



Implementation and Evaluation of Toeplitz Strong Extractor for Post-Processing of QRNGs on Various Hardware Systems



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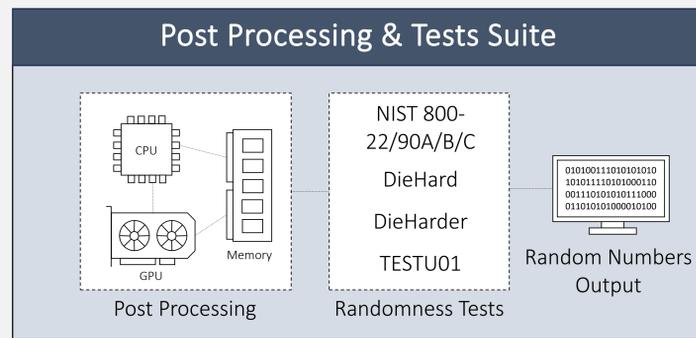
INTRODUCTION

Post Processing of QRNG Raw Data

- To distil Quantum Randomness from Quantum + Classical Noise
- To generate information-theoretically provable Quantum Random Numbers

Post Processing Methods

- Matrix Multiplication based Toeplitz Hashing
- Fast Fourier Transform based Toeplitz Hashing



Hardware System Specifications		
Processor	AMD Ryzen 5 4600H 6C 12T	Intel Xeon Gold 6226R 16C 32T
Graphics Card	Nvidia GTX1650(M) 4 GB	Nvidia Quadro GV100 32 GB
RAM	DDR4 3200MHz 16GB	DDR3 1866 MHz 64 GB

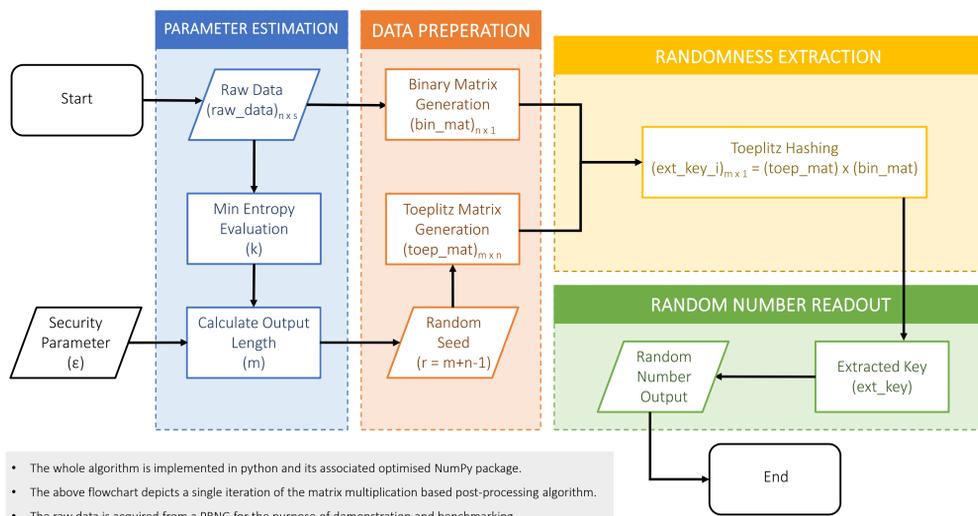
Algorithmic Schematic for QRNG Post-Processing



Ma X et al., Physical Review A, 87(6), 062327, (2013)

ALGORITHMIC FLOW

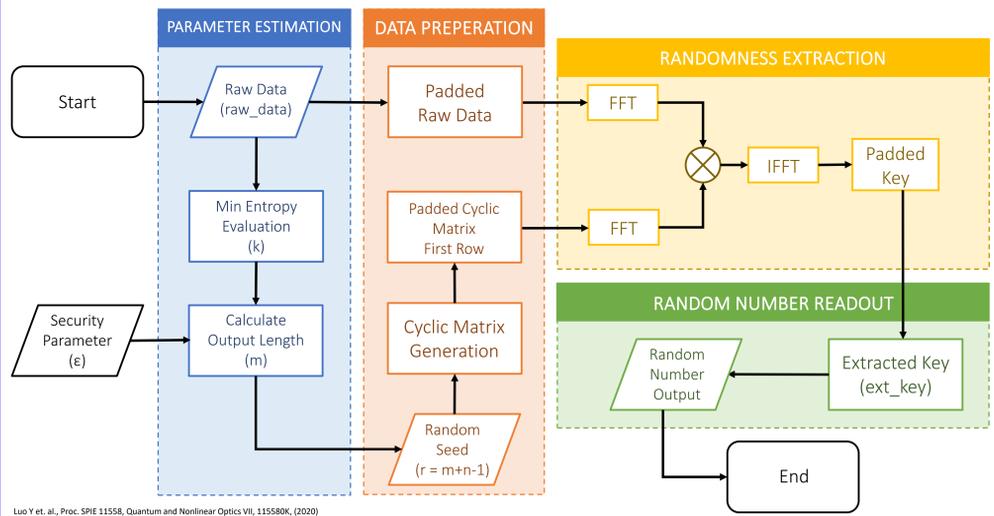
Matrix Multiplication based Toeplitz Hashing



- The whole algorithm is implemented in python and its associated optimised NumPy package.
- The above flowchart depicts a single iteration of the matrix multiplication based post-processing algorithm.
- The raw data is acquired from a PRNG for the purpose of demonstration and benchmarking.
- The output length is calculated in accordance to the Leftover Hash Lemma.

Zhang X et al., IEEE-NPSS Real Time Conference (RTL), Padua, Italy, pp. 1-5, (2016)

Fast Fourier Transform based Toeplitz Hashing

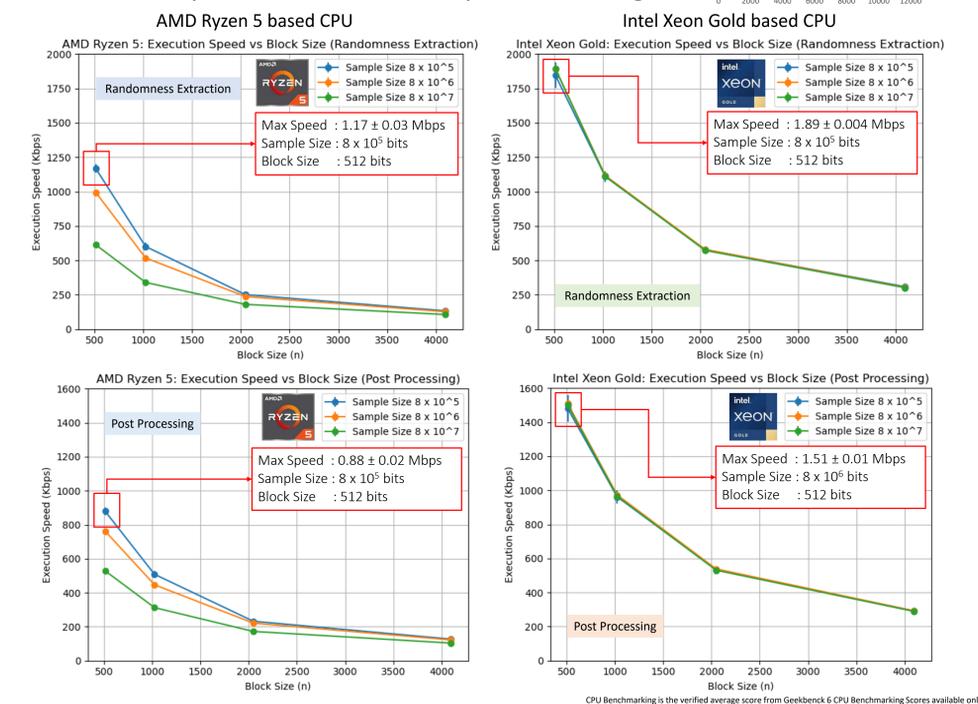


Luo Y et al., Proc. SPIE 11558, Quantum and Nonlinear Optics VII, 115580K, (2020)

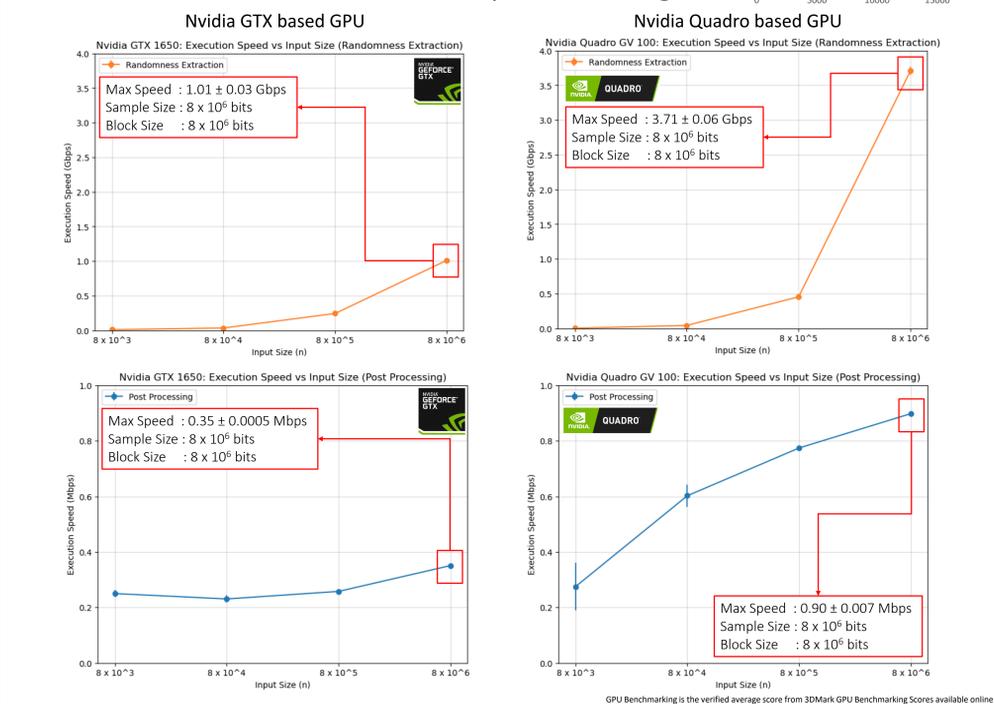
- The operation of matrix multiplication is converted to fast convolution based on the properties of the cyclic matrix.
- The GPU based FFT implementation used is highly optimized for 2^n inputs due to which we pad both the raw data and the first row of the cyclic matrix to the closest 2^n number.

BENCHMARKING AND RESULTS

Matrix Multiplication based Toeplitz Hashing



Fast Fourier Transform Based Toeplitz Hashing



CONCLUSION

- We systematically elucidate the various implementation steps of the post-processing algorithm for the QRNG system.
- We have significantly improved the time and space complexity of the randomness extraction step by moving from matrix multiplication based Toeplitz Hashing to FFT based implementation of Toeplitz Hashing.
- We provide a detailed comparison of various implementation methods on different hardware systems, achieving a maximum speed of 3.7 Gbps for Randomness Extraction in a single iteration on Nvidia Quadro GV100 GPU.

Method Used	Time Complexity	Space Complexity	Hardware (CPU / GPU)	Sample Size (l x s)	Block Size (n)	Speed of Execution
Matrix Multiplication	$O(n^2)$	$O(n * m)$	AMD Ryzen 5 4600H	8×10^5	512	1.17 Mb/s
			Intel Xeon Gold 6226R	8×10^7	512	1.89 Mb/s
Fast Fourier Transform	$O(n \cdot \log_2 n)$	$O(n+m-1)$	Nvidia GTX1650 (M)	8×10^6	8×10^6	1.01 Gbps
			Nvidia Quadro GV100	8×10^6	8×10^6	3.71 Gbps